[2007/11/11]

TEST IDEALS AND MULTIPLIER IDEALS

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1. CHARACTERISTIC $p > 0$ INVARIANTS

Problem 1.05. *Given an ideal I of $\mathbb{Z}[x]$, the HSL numbers are the "indices of nilpotency" of the action of Frobenius on $H^{top}_{(\underline{x})}\left(\mathbb{Z}_p[\underline{x}]/I_p\right)$, for primes p. Is the limsup of the HSL numbers 1?*

Problem 1.1. *Realize effective computations of numerical F-invariants.*

Problem 1.15. *Do Howald's nondegeneracy conditions with respect to the Newton polyhedron on the term ideal provide conditions under which* fpt(*polynomial*) = fpt(*associated term ideal*)*?*

Problem 1.2. *For a graded sequence of ideals I_m of a ring R, does* $\lim_{m\to\infty} m \cdot \mathrm{fpt}(I_m)$ *exist?*

Remark. This is known for log canonical thresholds.

Problem 1.25. *Investigate the existence and rationality of F-thresholds.*

Remark. Existence is known for a ring that is F-pure on the punctured spectrum.

Problem 1.3. *Investigate the discreteness and rationality of F-jumping numbers in the non-$\mathbb{Q}$-Gorenstein case.*

Problem 1.35. *Fix d. Does there exist a sequence of singular algebras $\{R_n\}$ over a fixed finite field of characteristic $p > 0$, and of dimension d, such that the $e_{HK}(R_n)$ descend to some number α_d from above?*

Problem 1.4. *Fix d. What is the maximum value of the F-pure threshold of $\mathfrak{m}$ of a non-$\mathbb{Q}$-Gorenstein local ring $(R, \mathfrak{m})$ of dimension d?*

Problem 1.45. *Given a determinantal ring R, compute $e_{HK}(R)$ and $s(R)$.*

Problem 1.5. *Investigate the upper semicontinuity of the map $\mathfrak{p} \in \operatorname{Spec} R \mapsto e_{HK}(R_{\mathfrak{p}})$.*

Problem 1.55. *Investigate the lower semicontinuity of the map* $\mathfrak{p} \in \operatorname{Spec} R \mapsto s(R_{\mathfrak{p}})$.

Problem 1.6. *If* a_e *denote of* F*-splitting numbers of a strongly* F*-regular local ring* R *(i.e.,* $R^{1/p^e} \cong R^{a_e} \oplus M$ *as* R*-modules, and* M *contains no free* R*-module summands), then is* $a_e = s(R)p^{ed} + Cp^{e(d-1)} + O\left(p^{e(d-2)}\right)$ *for some constant* C*?*

Remark. In the Gorenstein case, it is known and C=0.

Problem 1.65. *For* $(R, \mathfrak{m})$ *a local, normal domain of characteristic* $p > 0$ *and dimension* d*, let* μ_e *denote the minimal number of generators of* $\operatorname{Hom}_R(R^{1/p^e}, R)$ *as an* R*-module. Does* $\lim_{e\to\infty} \frac{\mu_e}{p^{ed}}$ *exist? If so, what is it?*

Remark. In the $\mathbb{Q}$-Gorenstein case, it exists and equals $e_{HK}R$.

Problem 1.7. *For* $(R, \mathfrak{m})$ *a local, normal domain of characteristic* $p > 0$*, let* x *be a minimal generator of* $\mathfrak{m}$*, and let* v *denote a* t^{th} *root of* x*. Relate* $e_{HK}(R)$ *and* $e_{HK}(R[v])$.

2. Other and Related Problems

Problem 2.05. *Generalize the Hara-Yoshida Theorem on restriction of (Hacon-de Fernex) multiplier ideals to test ideals to the non-*$\mathbb{Q}$*-Gorenstein case. Can this be done in the numerically Gorenstein setting?*

Problem 2.1. *Investigate the subadditivity of (Hacon-de Fernex) multiplier ideals in the non-*$\mathbb{Q}$*-Gorenstein case.*

Remark. This is closely related to problem 2.05.

Problem 2.15. *Define a "test ideal" that relates to the Mather discrepancy multiplier ideal, in the non-*$\mathbb{Q}$*-Gorenstein case.*

Problem 2.2. *Investigate lifting sections (for cohomology) using test ideals.*

Problem 2.25. *Given ideals* $I_1, \ldots, I_n$ *of a regular,* F*-finite ring* R*, consider the map sending an* r*-tuple of nonnegative real numbers* $(\lambda_1, \ldots, \lambda_r)$ *to* $\tau\left({I_1}^{\lambda_1} \cdot \ldots \cdot {I_n}^{\lambda_n}\right)$*, the mixed test ideal. If we bound all the* λ_i *by some fixed* M*, does there exist a rational polyhedral decomposition such that the function described is constant on the interior of each region?*

Problem 2.3. *Realize effective computations of test ideals.*

Direct summand conjecture

Problem 2.35. *Prove/disprove the Direct Summand Conjecture.*

Problem 2.4. *Extend tight closure to mixed characteristic.*

Problem 2.5. *Given a smooth, projective variety X over a number field, does there exist a dense set of primes for which the action of Frobenius on the coherent cohomology $H^i(X_p, \mathcal{O}_{X_p})$ is not nilpotent?*

Problem 2.45. *Suppose that R is a local, equidimensional, F-pure ring of dimension d with embedding dimension n. Is the multiplicity of R at most $\binom{n}{d}$?*

Problem 2.55. *Do determinantal rings have finite F-representation type?*

Problem 2.6. *For $p \geq 11$, does $\mathbb{F}_p[x, y, z]/(x^2 + y^3 + z^7)$ have finite F-representation type?*

Problem 2.65. *Suppose that R is a local ring and $f \in R$ is a regular element such that $R/(f)$ is F-injective. Does this imply that R is F-injective?*

Problem 2.7. *Given a log pair (X, Δ), we know that there exists a finite map $\phi : Y \to X$ such that $\operatorname{im}\left(\phi_* \mathcal{O}_Y\left(K_Y - \phi^*\left(K_X + \Delta\right)\right) \xrightarrow{trace} \mathcal{O}_X\right) = \tau(X, \Delta)$. Is this statement true if further decorated by α^t? What about for alterations?*

Problem 2.75. *Identify a possible positive characteristic analog of minimal log discrepancy.*

Problem 2.8. *Investigate possible Bertini theorems for F-singularities.*

Problem 2.85. *Investigate the applications of the topics of the conference (e.g., test ideals) to projective geometry.*

REFERENCES