

[2007/11/11]

GEOMETRY OF LARGE NETWORKS

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1. PROBLEM SESSION

This (below) was an attempt to record the problem session (Tuesday, November 1st), the way it proceeded. Your additions and comments are welcome.

- Dan

Problem 1.05. *How do we measure the curvature of real networks?*

Problem 1.3. *What is the curvature of a network? How to define it?*

Problem 1.15. *Why measure it? (What are the real-world applications?)*

Remark. [Weinberger] Wasn't that your result, that the internet has curvature -1 and dimension 2 ?

Remark. [Krioukov] So, yes, we need more examples like that, showing the significance of curvature.

Problem 1.2. *Find (different) notions of distance on the space of graphs, or perhaps measures on this space. In particular: how far is a given graph from being a tree?*

Remark. [Holmes] perhaps need to look at non-dense graphs; I have the impression that the problem is solved for dense ones.

Remark. [Gao] How about tree-width?

Remark. [Sullivan] Tree-width is not good enough. There is a relation to tree-length.

Problem 1.25. *Is curvature related to clustering?*

Remark. [Baryshnikov] What is meant by clustering?

Problem 1.55. *Is there a scale at which the structure of a (real) network "looks like" a manifold? (e.g., Krioukov's result)*

Remark. [Krioukov] This relates to the (previous) questions about how to define hyperbolicity or curvature for a *single* finite graph.

Problem 1.35. *For random graphs, is there an idea for a degree of heterogeneity that could indicate hyperbolicity*

Remark. [Weinberger] Similar to the question of frequency of hyperbolic groups among finitely generated/presented groups: could ask –

* What is a set of conditions ensuring hyperbolicity of an ensemble of random graphs (i.e., the set of hyperbolic graphs in the ensemble has probability one)?

* (Mahoney) important to emphasize randomness.

Problem 1.4. *What would be meant by a graph having *positive* curvature? (Grids don't have it; trees don't; how about spheres?)*

Remark. [Baryshnikov, Weinberger] That depends on a choice of scale.

Problem 1.45. *What precise notions of negative curvature imply congestion?*

Remark. [Weinberger] What price are you willing to pay to avoid congestion?

Remark. [Bonahon/Saniee] *Where* does congestion occur? (something analogous to convex core?)

Problem 1.5. *What are the data required to define congestion?*

Remark. [Holmes] Do weights on the network or some such affect this?

Problem 1.1. *Need ways to compute geodesics on spaces where curvature is variable (using embedded graphs).*

REFERENCES